

### Our Purpose:

By using the methods contained in this policy and by using concrete, pictorial and abstract representations and structures to deepen understanding along the learning journey, children develop their secure understanding of the four standard written methods by Year 6 – written addition, subtraction, short and long multiplication, and short and long division.

All staff working with children must have their own secure understanding of the methods and representations used at Saltaire Primary School through each stage of their learning journey so that children are not confused by alternative or inappropriate methods.

This clear, robust, consistent and evidenced approach will ensure that children are able to master the key concepts of mathematics with fluency, reasoning and problem solving skills for life.

### Our Ethos

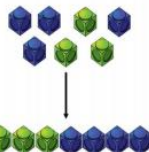

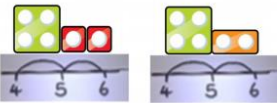
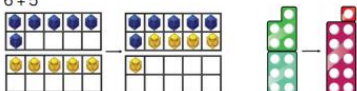
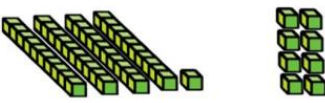
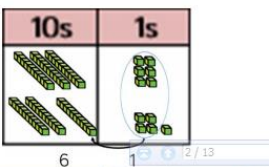
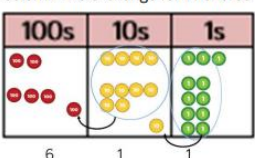
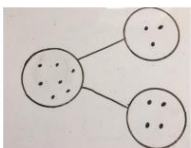
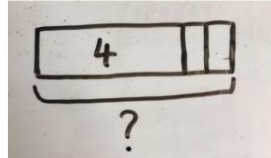
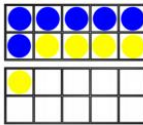
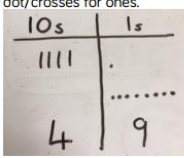
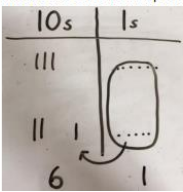
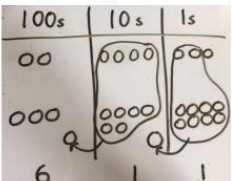
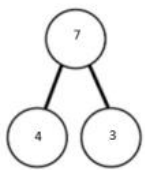

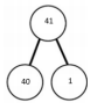
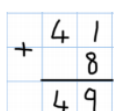
- **Maths teaching for mastery rejects the idea that a large proportion of people ‘just can’t do maths’.**
- **All pupils are encouraged by the belief that by working hard at maths they can succeed.**
- Pupils are taught through whole-class interactive teaching, where the focus is on all pupils working together on the same lesson content at the same time, as happens in Shanghai and several other regions that teach maths successfully. This ensures that all can master concepts before moving to the next part of the curriculum sequence, allowing no pupil to be left behind.
- If a pupil fails to grasp a concept or procedure, this is identified quickly and early intervention ensures the pupil is ready to move forward with the whole class in the next lesson.
- Lesson design identifies the new mathematics that is to be taught, the key points, the difficult points and a carefully sequenced journey through the learning.
- Procedural fluency and conceptual understanding are developed in tandem because each supports the development of the other.
- It is recognised that practice is a vital part of learning, but the practice used is intelligent practice that both reinforces pupils’ procedural fluency and develops their conceptual understanding.
- Significant time is spent developing deep knowledge of the key ideas that are needed to underpin future learning. The structure and connections within the mathematics are emphasised, so that pupils develop deep learning that can be sustained.
- Key facts such as multiplication tables and addition facts within 10 are learnt to automaticity to avoid cognitive overload in the working memory and enable pupils to focus on new concepts.

(NCETM, *The Essence of Maths Teaching for Mastery*, June 2016)

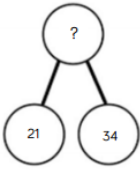
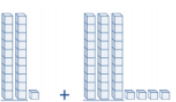
Table of Contents:

1	Addition: NC Objectives
	Addition: Representation and Structure
2	Subtraction: NC Objectives
	Subtraction: Representation and Structure
3	Multiplication: NC Objectives
	Multiplication: Representation and Structure
4	Division: NC Objectives
	Division: Representation and Structure
5	Fractions: Objectives and Progression
6	Problem Solving: Progression
Appendix 1: National Curriculum Guidance for formal written methods	
Appendix 2: Swap Shop Method	
Appendix 3: Key Objectives by Year Group	
Appendix 4: Overview and Guidance	

	Addition: Objectives					
Year 1	Read, write and interpret mathematical statements involving addition (+) and equals (=) signs (using numbers from 0 to 20).	Add 1 and 2 digit numbers to 20, including zero.	Solve one-step problems that involve addition, using concrete objects and pictorial representations, and missing number problems such as $7 = \square + 9$ .	.		
Year 2	Recall and use addition facts to 20 fluently, and derive and use related facts up to 100.	Add numbers using concrete objects, pictorial representations, and mentally, including: □ a two-digit number and ones	a two-digit number and tens two two-digit numbers adding three one-digit numbers (for all, without going across a boundary of 10 or 100).	Show that addition can be done in any order (commutative).	Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.	
Year 3	Add numbers with up to three digits, using formal written method of columnar addition.	Estimate the answer to a calculation and use inverse operations to check answers.	Solve problems, including missing number problems, using number facts, place value and more complex addition and subtraction.			
Year 4	Add numbers with up to 4 digits using the formal written methods of columnar addition	Estimate and use inverse operations to check answers to a calculation.	Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.			
Year 5	Add whole numbers with more than 4 digits, including using formal written methods (columnar addition).	Add numbers mentally with increasingly large numbers.	Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy.	Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.		
Year 6	Use their knowledge of the order of operations to carry out calculations involving the four operations.	Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.	Solve problems involving addition, subtraction, multiplication and division.	Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.		

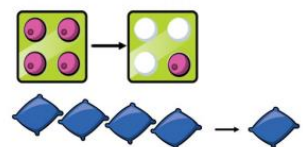
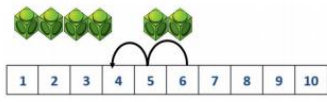
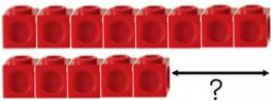
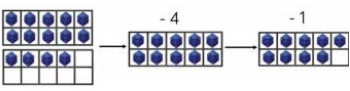
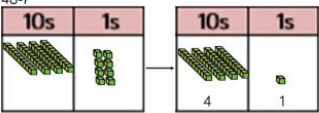
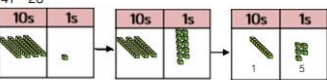
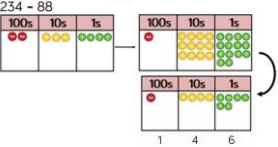
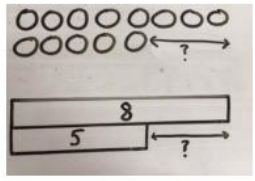
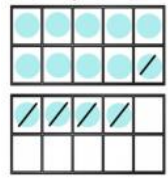
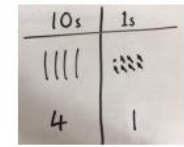
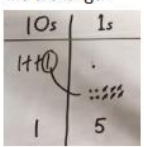
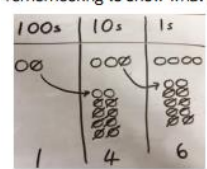
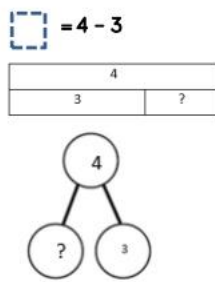
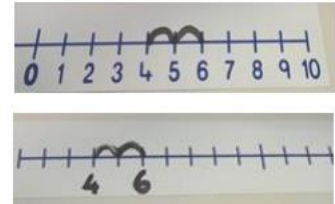
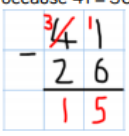
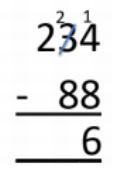
Addition: Representation and Structure				
Concrete	Pictorial	Abstract		
<p><b>Combining two parts to make a whole</b> (use other resources too e.g. eggs, shells, teddy bears, cars).</p>  <p>Counting on using number lines using cubes or Numicon.</p>   <p><b>Regrouping to make 10;</b> using ten frames and counters/cubes or using Numicon.</p> <p>6 + 5</p>  <p><b>TO + O using base 10.</b> Continue to develop understanding of partitioning and place value.</p> <p>41 + 8</p>  <p><b>TO + TO using base 10.</b> Continue to develop understanding of partitioning and place value.</p> <p>36 + 25</p>  <p><b>Use of place value counters to add HTO + TO, HTO + HTO etc.</b> When there are 10 ones in the 1s column- we exchange for 1 ten, when there are 10 tens in the 10s column- we exchange for 1 hundred.</p> 	<p>Children to represent the cubes using dots or crosses. They could put each part on a part whole model too.</p>  <p>A bar model which encourages the children to count on, rather than count all.</p>  <p>Children to draw the ten frame and counters/cubes.</p>  <p>Children to represent the base 10 e.g. lines for tens and dot/crosses for ones.</p>  <p>Children to represent the base 10 in a place value chart.</p>  <p>Children to represent the counters in a place value chart, circling when they make an exchange.</p> 	<p><math>4 + 3 = 7</math> Four is a part, 3 is a part and the whole is seven.</p>  <p>The abstract number line: What is 2 more than 4? What is the sum of 2 and 4? What is the total of 4 and 2? <math>4 + 2</math></p>  <p>Children to develop an understanding of equality e.g.</p> <p><math>6 + \square = 11</math> <math>6 + 5 = 5 + \square</math> <math>6 + 5 = \square + 4</math></p> <p><math>41 + 8</math></p>  <p><math>1 + 8 = 9</math> <math>40 + 9 = 49</math></p>  <p>Looking for ways to make 10.</p> <p><math>36 + 25 =</math></p> <p>30 + 20 = 50 5 + 5 = 10 50 + 10 + 1 = 61</p> <p>1 5 36 243</p> <p>Formal method:</p> <table> <tr> <td><math>\begin{array}{r} +25 \\ 36 \\ \hline 61 \\ 1 \end{array}</math></td> <td><math>\begin{array}{r} 243 \\ +368 \\ \hline 611 \\ 1 \end{array}</math></td> </tr> </table>	$\begin{array}{r} +25 \\ 36 \\ \hline 61 \\ 1 \end{array}$	$\begin{array}{r} 243 \\ +368 \\ \hline 611 \\ 1 \end{array}$
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## Conceptual variation; different ways to ask children to solve $21 + 34$

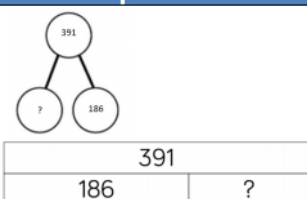
 <table border="1" data-bbox="199 1890 448 1937"><tr><td colspan="2">?</td></tr><tr><td>21</td><td>34</td></tr></table>	?		21	34	<p>Word problems:</p> <p>In year 3, there are 21 children and in year 4, there are 34 children. How many children in total?</p> <p><math>21 + 34 = 55</math>. Prove it</p>	$\begin{array}{r} 21 \\ +34 \\ \hline \end{array}$ <p><math>21 + 34 =</math></p> <p><span style="border: 1px dashed black; padding: 2px 10px;"> </span> <math>= 21 + 34</math></p> <p>Calculate the sum of twenty-one and thirty-four.</p>	 <p>Missing digit problems:</p> <table border="1" data-bbox="1110 1874 1289 2000"><thead><tr><th>10s</th><th>1s</th></tr></thead><tbody><tr><td></td><td></td></tr><tr><td></td><td>?</td></tr><tr><td>?</td><td>5</td></tr></tbody></table>	10s	1s				?	?	5
?															
21	34														
10s	1s														
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?	5														

Subtraction: Objectives				
Year 1	Read, write and interpret mathematical statements involving subtraction (-) and equals (=) signs (using numbers from 0 to 20) Represent and use number bonds and related subtraction facts within 20.	Subtract 1 and 2 digit numbers to 20, including zero. Solve one-step problems that involve subtraction, using concrete objects and pictorial representations, and missing number problems.		
Year 2	Recall and use subtraction facts to 20 fluently, and derive and use related facts up to 100	subtract numbers using concrete objects, pictorial representations, and mentally, including: a two-digit number and ones a two-digit number and tens two two-digit numbers adding three one-digit numbers (for all, without going across a boundary of 10 or 100)	Show that subtraction of one number from another cannot be done in any order.	Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.
Year 3	Subtract numbers with up to three digits, using formal written method of columnar subtraction (where the smaller number contains digits greater than 5 so decomposition has to occur)	Estimate the answer to a calculation and use inverse operations to check answers.	Solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.	
Year 4	Subtract numbers with up to 4 digits using the formal written methods of columnar subtraction where appropriate	Estimate and use inverse operations to check answers to a calculation	Solve subtraction two-step problems in contexts, deciding which operations and methods to use and why.	
Year 5	Subtract numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)	Subtract numbers mentally with increasingly large numbers	Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy	Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.
Year 6	Use their knowledge of the order of operations to carry out calculations involving the four operations	Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why	Solve problems involving addition, subtraction, multiplication and division	Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy



Subtraction: Representation and Structure		
Concrete	Pictorial	Abstract
<p><b>Physically taking away and removing objects from a whole</b> (ten frames, Numicon, cubes and other items such as beanbags could be used).</p> <p><math>4 - 3 = 1</math></p>  <p><b>Counting back</b> (using number lines or number tracks) children start with 6 and count back 2.</p> <p><math>6 - 2 = 4</math></p>  <p><b>Finding the difference</b> (using cubes, Numicon or Cuisenaire rods, other objects can also be used).</p> <p>Calculate the difference between 8 and 5.</p>  <p><b>Making 10</b> using ten frames.</p> <p><math>14 - 5</math></p>  <p><b>Column method</b> using base 10.</p> <p><math>48 - 7</math></p>  <p><b>Column method</b> using base 10 and having to exchange.</p> <p><math>41 - 26</math></p>  <p><b>Column method</b> using place value counters.</p> <p><math>234 - 88</math></p> 	<p>Children to draw the cubes/other concrete objects which they have used or use the bar model to illustrate what they need to calculate.</p>  <p>Children to present the ten frame pictorially and discuss what they did to make 10.</p>  <p>Children to represent the base 10 pictorially.</p>  <p>Represent the base 10 pictorially, remembering to show the exchange.</p>  <p>Represent the place value counters pictorially, remembering to show what has been exchanged.</p> 	<p><math>4 - 3 =</math></p>  <p>Children to represent the calculation on a number line or number track and show their jumps. Encourage children to use an empty number line</p>  <p>Formal column method. Children must understand that when they have exchanged the 10 they still have 41 because <math>41 = 30 + 11</math>.</p>  <p>Formal column method. Children must understand what has happened when they have crossed out digits.</p> 

## Conceptual variation; different ways to ask children to solve $391 - 186$



Raj spent £391, Timmy spent £186.  
How much more did Raj spend?

Calculate the difference between 391 and 186.

$\square = 391 - 186$

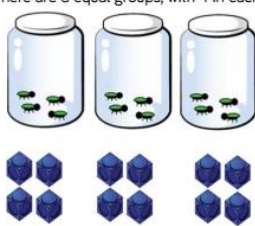
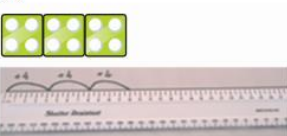
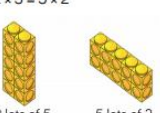
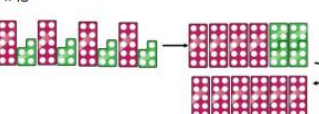
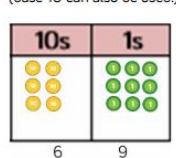
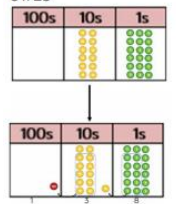
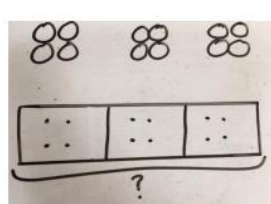
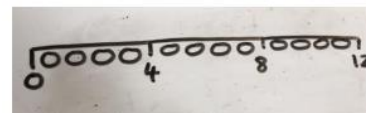
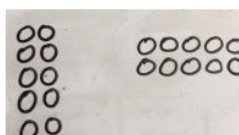
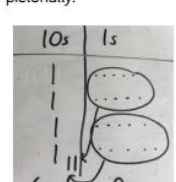
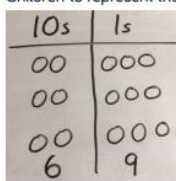
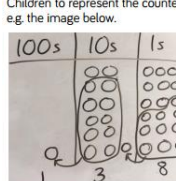
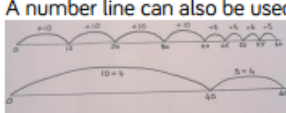
$391$   
 $-186$

What is 186 less than 391?

Missing digit calculations

$$\begin{array}{r} 39\square \\ - \square\square6 \\ \hline \square05 \end{array}$$

Multiplication: Objectives				
Year 1	Solve one-step problems involving multiplication by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher			
Year 2	Recall and use multiplication facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers	Calculate mathematical statements for multiplication within the multiplication tables and write them using the multiplication (×) and equals (=) signs	Show that multiplication of two numbers can be done in any order (commutative)	
Year 3	Recall and use multiplication and facts for the 3, 4 and 8 multiplication tables	Write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods		
Year 4	Recall multiplication for multiplication tables up to $12 \times 12$	Use place value, known and derived facts to multiply mentally, including: multiplying by 0 and 1; multiplying together three numbers	Multiply two-digit and three-digit numbers by a one-digit number using formal written layout	
Year 5	Solve problems involving multiplication where larger numbers are used by decomposing them into their factors	Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers	Multiply numbers mentally drawing upon known facts	Multiply whole numbers and those involving decimals by 10, 100 and 1000
Year 6	Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long Multiplication	Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy		

Multiplication: Representation and Structure		
Concrete	Pictorial	Abstract
<p><b>Repeated grouping/repeated addition</b>  <math>3 \times 4</math>  <math>4 + 4 + 4</math>            There are 3 equal groups, with 4 in each group.</p>  <p><b>Number lines to show repeated groups-</b>  <math>3 \times 4</math></p>  <p>Cuisenaire rods can be used too.</p> <p><b>Use arrays to illustrate commutativity</b> counters and other objects can also be used.  <math>2 \times 5 = 5 \times 2</math></p>  <p>2 lots of 5      5 lots of 2</p> <p><b>Partition to multiply</b> using Numicon, base 10 or Cuisenaire rods.  <math>4 \times 15</math></p>  <p><b>Formal column method</b> with place value counters (base 10 can also be used.) <math>3 \times 23</math></p>  <p><b>Formal column method</b> with place value counters.  <math>6 \times 23</math></p> 	<p>Children to represent the practical resources in a picture and use a bar model.</p>  <p>Represent this pictorially alongside a number line e.g.:</p>  <p>Children to represent the arrays pictorially.</p>  <p>Children to represent the concrete manipulatives pictorially.</p>  <p>Children to represent the counters pictorially.</p>  <p>Children to represent the counters/base 10, pictorially e.g. the image below.</p> 	<p><math>3 \times 4 = 12</math>  <math>4 + 4 + 4 = 12</math></p> <p>Children to be able to use an array to write a range of calculations e.g.</p> <p><math>10 = 2 \times 5</math>  <math>5 \times 2 = 10</math>  <math>2 + 2 + 2 + 2 + 2 = 10</math>  <math>10 = 5 + 5</math></p> <p>Children to be encouraged to show the steps they have taken.</p> <p><math>4 \times 15</math>  <math>10 \quad 5</math></p> <p><math>10 \times 4 = 40</math>  <math>5 \times 4 = 20</math>  <math>40 + 20 = 60</math></p> <p>A number line can also be used</p>  <p>Children to record what it is they are doing to show understanding.</p> <p><math>3 \times 23</math>      <math>3 \times 20 = 60</math>  <math>20 \quad 3</math>      <math>3 \times 3 = 9</math>  <math>60 + 9 = 69</math></p> <p><math>23</math>  <math>\times 3</math>  <math>69</math></p> <p>Formal written method</p> <p><math>6 \times 23 =</math></p> <p><math>23</math>  <math>\times 6</math>  <math>138</math>  <math>1 \quad 1</math></p> <p><math>1 \quad 2 \quad 4</math>  <math>\times \quad 2 \quad 6</math>  <math>7 \quad 4 \quad 4</math>  <math>2 \quad 4 \quad 8 \quad 0</math>  <math>3 \quad 2 \quad 2 \quad 4</math>  <math>1 \quad 1</math>  <b>Answer: 3224</b></p>

## Conceptual variation; different ways to ask children to solve $6 \times 23$

23	23	23	23	23	23
?					

Mai had to swim 23 lengths, 6 times a week.  
 How many lengths did she swim in one week?

With the counters, prove that  $6 \times 23 = 138$

Find the product of 6 and 23

$6 \times 23 =$

$6 \times 23 =$

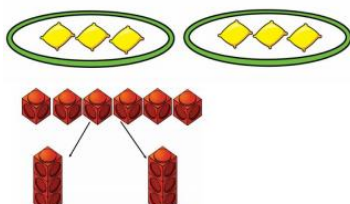
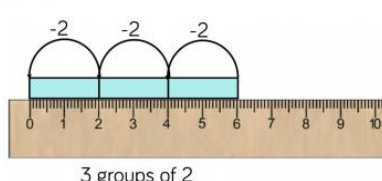

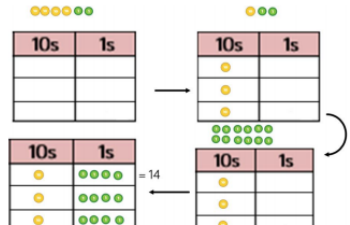
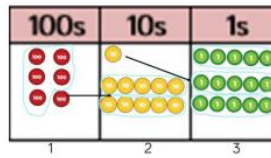
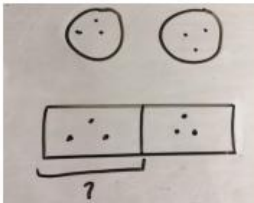
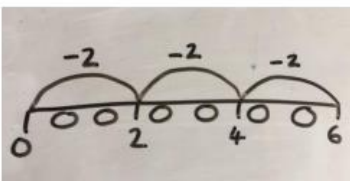
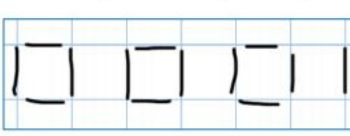
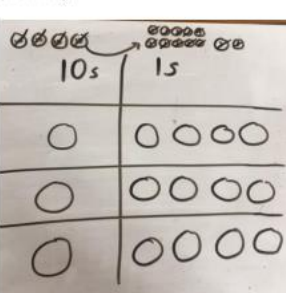
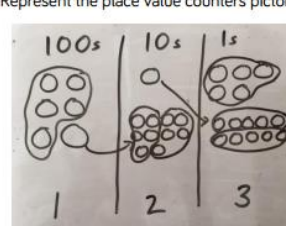
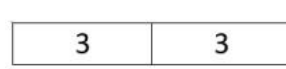
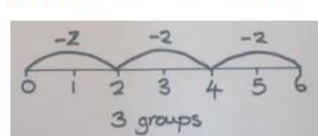
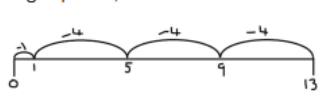
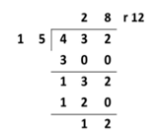
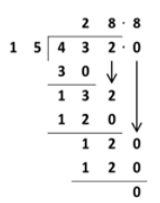
$6 \quad 23$   
 $\times 23 \quad \times 6$   
 ——— ———

What is the calculation?  
 What is the product?

100s	10s	1s
	20	18



Division: Objectives				
Year 1	Solve one-step problems involving division by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.			
Year 2	Recall and use division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers.	Calculate mathematical statements for division within the multiplication tables and write them using the division and equals (=) signs.	Show that division of one number by another cannot be done in any order.	
Year 3	Recall and use division facts for the 3, 4 and 8 multiplication tables.	Write and calculate mathematical statements for division using the multiplication tables that they know using mental and progressing to formal written methods.		
Year 4	Recall division facts for multiplication tables up to $12 \times 12$ .	Use place value, known and derived facts to divide mentally, including: dividing by 1.	Recognise and use factor pairs and commutatively in mental calculation.	
Year 5	Solve problems involving division where larger numbers are used by decomposing them into their factors.	Divide numbers up to 4-digits by a 1-digit number using the formal written method of short division and interpret the remainders appropriately for the context.	Divide numbers mentally drawing upon known facts.	Divide whole numbers and those involving decimals by 10, 100 and 1000.
Year 6	Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy.	Divide numbers up to 4-digits by a 2-digit whole number using the formal written method of long division and interpret remainders as whole number remainders, fractions, or by rounding as appropriate for the context		

Division: Representation and Structure		
Concrete	Pictorial	Abstract
<p><b>Sharing</b> using a range of objects. <math>6 \div 2</math></p>  <p><b>Repeated subtraction</b> using Cuisenaire rods above a ruler. <math>6 \div 2</math></p>  <p><b>2d + 1d with remainders</b> using lollipop sticks. Cuisenaire rods, above a ruler can also be used. <math>13 \div 4</math></p> <p>Use of lollipop sticks to form wholes- squares are made because we are dividing by 4.</p>  <p>There are 3 whole squares, with 1 left over.</p> <p><b>Sharing using place value counters.</b> <math>42 \div 3 = 14</math></p>  <p><b>Short division</b> using place value counters to group. <math>615 \div 5</math></p>  <ol style="list-style-type: none"> <li>1. Make 615 with place value counters.</li> <li>2. How many groups of 5 hundreds can you make with 6 hundred counters?</li> <li>3. Exchange 1 hundred for 10 tens.</li> <li>4. How many groups of 5 tens can you make with 11 ten counters?</li> <li>5. Exchange 1 ten for 10 ones.</li> <li>6. How many groups of 5 ones can you make with 15 ones?</li> </ol>	<p>Represent the sharing pictorially.</p>  <p>Children to represent repeated subtraction pictorially.</p>  <p>Children to represent the lollipop sticks pictorially.</p>  <p>There are 3 whole squares, with 1 left over.</p> <p>Children to represent the place value counters pictorially.</p>  <p>Represent the place value counters pictorially.</p> 	<p><math>6 \div 2 = 3</math></p>  <p>Children should also be encouraged to use their 2 times tables facts.</p> <p>Abstract number line to represent the equal groups that have been subtracted.</p>  <p><math>13 \div 4 = 3 \text{ remainder } 1</math></p> <p>Children should be encouraged to use their times table facts; they could also represent repeated addition on a number line.</p> <p>'3 groups of 4, with 1 left over'</p>  <p>Children to be able to make sense of the place value counters and write calculations to show the process.</p> <p><math>42 \div 3</math>  <math>42 = 30 + 12</math>  <math>30 \div 3 = 10</math>  <math>12 \div 3 = 4</math>  <math>10 + 4 = 14</math></p> <p>Children to the calculation using the short division scaffold.</p> <div style="text-align: center;"> <math display="block">\begin{array}{r} 123 \\ 5 \overline{) 615} \end{array}</math> </div> <div style="display: flex; justify-content: space-around;"> <div> <p><math>432 \div 15</math> becomes</p>  <p>Answer: 28 remainder 12</p> </div> <div> <p><math>432 \div 15</math> becomes</p>  <p>Answer: 28-8</p> </div> </div>

### Conceptual variation; different ways to ask children to solve $615 \div 5$

Using the part whole model below, how can you divide 615 by 5 without using short division?



I have £615 and share it equally between 5 bank accounts. How much will be in each account?

615 pupils need to be put into 5 groups. How many will be in each group?

$$5 \overline{) 615}$$

$$615 \div 5 =$$

$$\square = 615 \div 5$$

What is the calculation?  
What is the answer?



Fractions: Objectives						
	Counting in fractional steps	Recognising fractions	Comparing fractions/decimals	Comparing decimals	Equivalence	Calculating
Year 1		<p>recognise, find and name a half as one of two equal parts of an object, shape or quantity</p> <p>recognise, find and name a quarter as one of four equal parts of an object, shape or quantity</p>				
Year 2	<i>Pupils should count in fractions up to 10, starting from any number and using the 1/2 and 2/4 equivalence on the number line (Non Statutory Guidance)</i>	<p>recognise, find, name and write fractions <math>\frac{1}{3}</math>, <math>\frac{1}{4}</math>, <math>\frac{2}{4}</math> and <math>\frac{3}{4}</math> of a length, shape, set of objects or quantity</p>				
Year 3	count up and down in tenths	<p>recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators</p> <p>recognise that tenths arise from dividing an object into 10 equal parts and in dividing one – digit numbers or quantities by 10.</p>	compare and order unit fractions, and fractions with the same denominators			<p>add and subtract fractions with the same denominator within one whole (e.g. <math>\frac{5}{7}</math> + <math>\frac{1}{7}</math> = <math>\frac{6}{7}</math>)</p>
Year 4	count up and down in hundredths	recognise that hundredths arise when dividing an object by one hundred and dividing tenths by ten		compare numbers with the same number of decimal places up to two decimal places	recognise and show, using diagrams, families of common equivalent fractions and decimals	add and subtract fractions with the same denominator
Year 5		recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents (appears also in Equivalence)	compare and order fractions whose denominators are all multiples of the same number	read, write, order and compare numbers with up to three decimal places	<p>identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths</p> <p>read and write decimal numbers as fractions (e.g. <math>0.71 = \frac{71}{100}</math>)</p> <p>recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents</p>	<p>add and subtract fractions with the same denominator and multiples of the same number</p> <p>recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements &gt; 1 as a mixed number</p>

Year 6					recognise the per cent symbol (%) and understand that per cent relates to “number of parts per hundred”, and write percentages as a fraction with denominator 100 as a decimal fraction	(e.g. $\frac{2}{5} + \frac{4}{5} = \frac{6}{5} = 1\frac{1}{5}$ )  multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams
			compare and order fractions, including fractions >1 identify the value of each digit in numbers given to three decimal places		use common factors to simplify fractions; use common multiples to express fractions in the same denomination associate a fraction with division and calculate decimal fraction equivalents (e.g. 0.375) for a simple fraction (e.g. $\frac{3}{8}$ ) recall and use equivalences between simple fractions, decimals and percentages, including in different contexts.	add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions  multiply simple pairs of proper fractions, writing the answer in its simplest form (e.g. $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$ )  divide proper fractions by whole numbers (e.g. $\frac{1}{3} \div 2 = \frac{1}{6}$ )

	Problem Solving Progression						
Year 1	Act it Out	Trial and Error	Find a Pattern	Draw a bar model			
Year 2	Act it Out	Trial and Error	Find a Pattern	Draw a bar model			
Year 3	Act it Out	Trial and Error	Find a Pattern	Draw a bar model	Working Backwards	List or table	
Year 4	Act it Out	Trial and Error	Find a Pattern	Draw a bar model	Working Backwards	List or table	
Year 5		Trial and Error	Find a Pattern	Draw a bar model	Working Backwards	List or table	Algebraic
Year 6		Trial and Error	Find a Pattern	Draw a bar model	Working Backwards	List or table	Algebraic



## Appendix 1 – National Curriculum Guidance

### Mathematics Appendix 1: Examples of formal written methods for addition, subtraction, multiplication and division

This appendix sets out some examples of formal written methods for all four operations to illustrate the range of methods that could be taught. It is not intended to be an exhaustive list, nor is it intended to show progression in formal written methods. For example, the exact position of intermediate calculations (superscript and subscript digits) will vary depending on the method and format used.

For multiplication, some pupils may include an addition symbol when adding partial products. For division, some pupils may include a subtraction symbol when subtracting multiples of the divisor.

#### Addition and subtraction

789 + 642 becomes

$$\begin{array}{r} 789 \\ + 642 \\ \hline 1431 \\ \hline 1 \quad 1 \end{array}$$

Answer: 1431

874 – 523 becomes

$$\begin{array}{r} 874 \\ - 523 \\ \hline 351 \end{array}$$

Answer: 351

932 – 457 becomes

$$\begin{array}{r} 8 \quad 12 \quad 1 \\ 932 \\ - 457 \\ \hline 475 \end{array}$$

Answer: 475

932 – 457 becomes

$$\begin{array}{r} 1 \quad 1 \\ 932 \\ - 457 \\ \hline 475 \end{array}$$

Answer: 475

#### Short multiplication

24 × 6 becomes

$$\begin{array}{r} 24 \\ \times 6 \\ \hline 144 \\ \hline 2 \end{array}$$

Answer: 144

342 × 7 becomes

$$\begin{array}{r} 342 \\ \times 7 \\ \hline 2394 \\ \hline 2 \quad 1 \end{array}$$

Answer: 2394

2741 × 6 becomes

$$\begin{array}{r} 2741 \\ \times 6 \\ \hline 16446 \\ \hline 4 \quad 2 \end{array}$$

Answer: 16 446

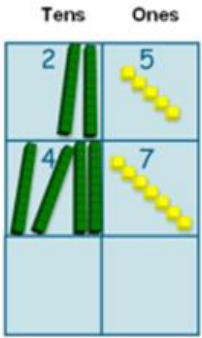
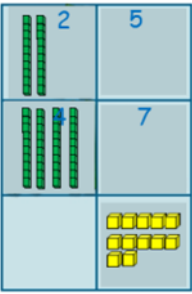

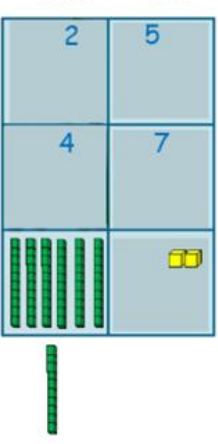


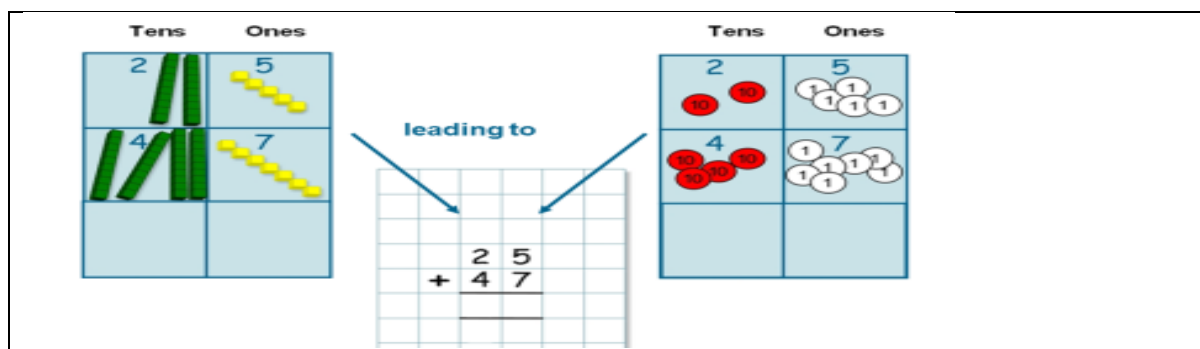
## Appendix 2 – Swap Shop method at Saltaire Primary School

### How to teach 'Swap shop' using bundles of straws or Base 10:

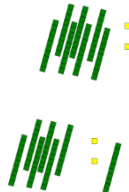
Addition: the children play a 'Swap shop' game with the teacher where they swap a ten stick for ten 'units' and vice versa. When the children have an understanding of the method shown, they practise on a prepared grid using concrete resources such as bundles and/ base 10/ and or counters. This supports with understanding the concepts that underpin the column addition method.

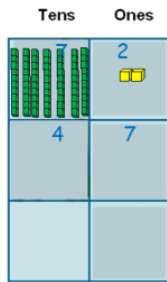


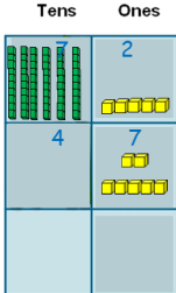
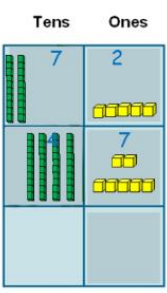
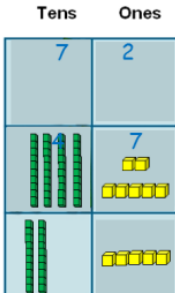
25+47 as a written method (reference to NCETM):

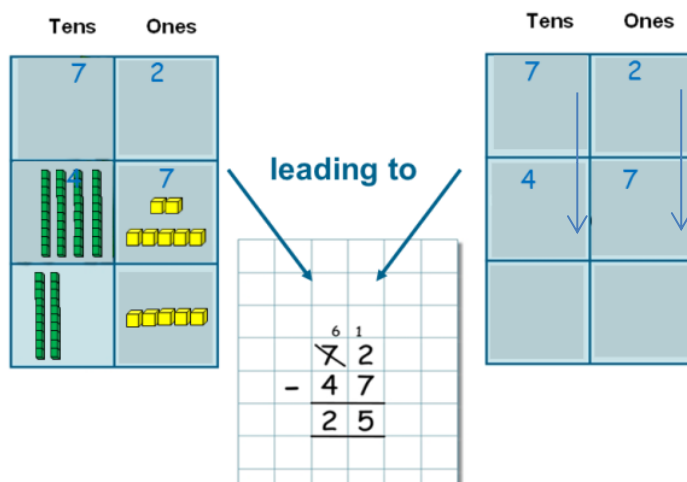
<u>Step One</u>	<u>Step Two</u>
	
<u>Step Three</u>	<u>Step Four</u>
	
<u>Step Five: the written method for addition</u>	



### Subtraction:

1. Start with the children playing a 'Swap shop' game with the teacher where they swap a ten stick for ten 'units' and vice versa then the children understanding the method shown and practised on a prepared grid using concrete resources such as bundles and/ base 10/ and or counters leading to the decomposition method that can be visualised:
 
2. Partitioning numbers in different ways using base 10 equipment as the resource needs to be explored:

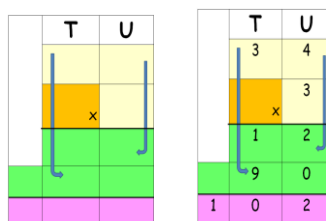
<u>Step One</u>	<u>Step Two</u>	<u>Step Three</u>
		
<u>Step Four</u>	<u>Step Five</u>	<u>Step Six</u>
		



### Teaching short multiplication using concrete resources

#### Expanded short multiplication

When beginning to teach short multiplication it is useful to give the answers separately first and then combine. So, multiply the digit in the Units column first and write the product underneath then multiply the tens digit and write the product underneath that. Finally total the two amounts. Use the following scaffold to help with the layout and use Base Ten resources to build as you go.



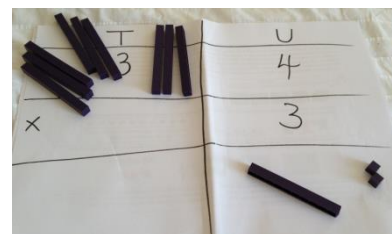
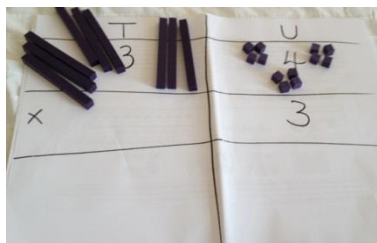
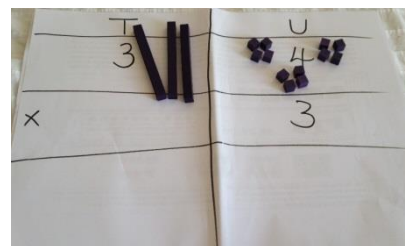
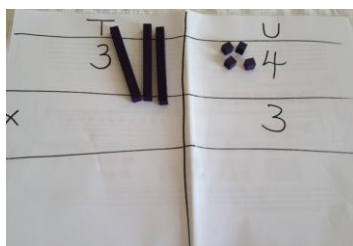
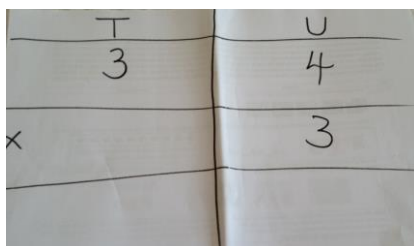
#### Standard Written Method of Short Multiplication

The following will demonstrate how this method can be taught to children with conceptual understanding related to place value.

*The following uses the example of  $34 \times 3$  ('thirty-four multiplied by three'; 'thirty-four, three times'):*

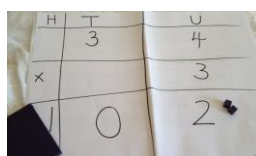
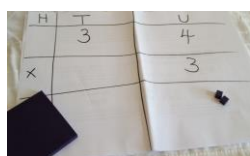
Draw a grid labeled with tens and ones and then build the number being multiplied (called the multiplicand) which is usually the larger amount of the two for ease:





Move the product of the two Units into the Units answer box: If that product exceeds 9 then it will need to be reorganised in relation to its place value and then 'carried' over:

Combine the product for the Tens column with the carried amount and consider if it needs to be 'carried' again (i.e. if the total of the carried amount and the product exceeds nine of them):



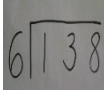
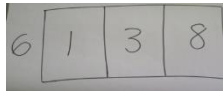
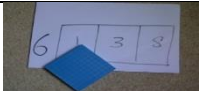
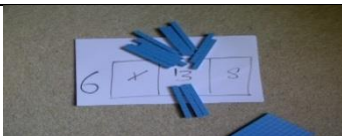
This

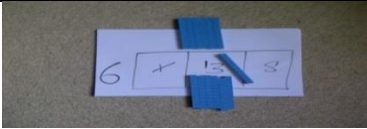
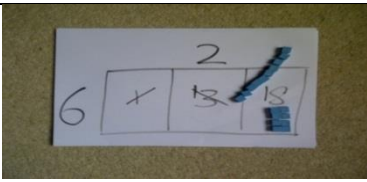
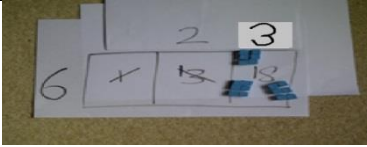
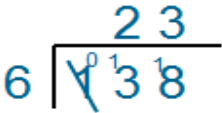
can be modelled and calculated using the grid here:

### How to teach short division:

The method could be demonstrated in a guided session using 'Base 10' or 'bundles of straws', describing that each of the pieces of equipment cannot be PHYSICALLY split into groups, as follows:  $138 \div 6$

	T	U
	3	4
x		3
1	0	2
	1	

When exploring how the method works, write the calculation so that the digits are separated:	 
Work through a section at a time being aware of the place value;	 How many groups of sixes can I physically break this 100 flat into? 'None.'
So move the digit not used across and then build the new number (which is now thought of as thirteen tens because it is in the tens column) with ten sticks:	

<p>'How many groups of six can you physically make out of thirteen tens?' 'Two groups of six tens each with one ten stick left over (remaining)':</p>		
<p>'Carry the remaining digit over to the next section and then build the new number':</p>		
<p>'How many groups of six can you make with eighteen units?' 'Three groups of six':</p>		
<p>'So the answer to one hundred and thirty-eight divided by six is twenty-three groups of six':</p>		

## Appendix 3: Key Objectives by Year Group

	Year One	Year Two	Year Three	Year Four	Year Five	Year Six
<b>Place value</b>	<p>Count, read and write numbers to 100 in numerals.</p> <p>Recognise the place value of each digit in a two-digit number (tens, ones).</p>	<p>Recognise the place value of each digit in a two-digit number (tens, ones). EXS</p> <p>Use place value and number facts to solve problems.</p>	<p>Recognise the value of each digit in a three-digit number</p> <p>Compare and order numbers</p> <p>Count in multiples up to 1000</p>	<p>Recognise the place value of each digit in a four-digit number (thousands, hundreds, tens, and ones).</p> <p>Order and compare numbers beyond 1000 and negative numbers.</p> <p>Round any number to the nearest 10, 100 or 1000.</p> <p>Compare numbers with the same number of decimal places up to two decimal places.</p>	<p>Read, write, order and compare numbers to at least 1 000 000 and determine the value of each digit.</p> <p>Count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000.</p> <p>Read, write, order and compare numbers with up to three decimal places.</p>	<p>Read, write, order and compare numbers up to 10 000 000 and determine the value of each digit.</p> <p>Solve number and practical problems that involve large numbers, rounding and negative numbers.</p>
<b>Addition and subtraction</b>	<p>Identify and represent numbers using objects and pictorial representations including the number line, and use the language of: equal to, more than, less than (fewer), most, least.</p> <p>Represent and use number bonds and related subtraction facts within 20</p> <p>Add and subtract one-digit and two-digit numbers to 20, including zero.</p>	<p>Add and subtract numbers using concrete objects, pictorial representations, and mentally, including two two-digit numbers.</p> <p>Two-digit number and ones, tens.</p> <p>Two two-digit numbers and add three one digit numbers.</p> <p>Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.</p>	<p>Add and subtract up to 3 digit numbers using column addition</p> <p>Add and subtract numbers mentally, including three-digit number and ones, tens and hundreds.</p> <p>Solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction</p>	<p>Add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate.</p> <p>Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why</p>	<p>Add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction).</p> <p>Add and subtract numbers mentally with increasingly large numbers.</p> <p>Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.</p>	<p>Perform mental calculations, including with mixed operations and large numbers.</p> <p>Use their knowledge of the order of operations to carry out calculations involving the four operations</p> <p>Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.</p> <p>Solve problems involving addition, subtraction, multiplication and division.</p>

Multiplication and division	Count in multiples of twos, fives and tens.	Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers.	Recall and use multiplication facts for the 3, 4 and 8 multiplication tables in addition to the 2, 3, 5 & 10 multiplication tables.	Recall multiplication and division facts for multiplication tables up to $12 \times 12$ .	Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers.	Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication.
		Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multi-plication and division facts, including problems in contexts.	Use formal written methods to calculate multiplications of two-digit numbers by one-digit numbers  Recall and use division facts for the 3, 4 and 8 multiplication tables in addition to the 2, 3, 5 & 10 multiplication tables  Use formal written methods to calculate divisions of two-digit numbers by one-digit numbers	Multiply two-digit and three-digit numbers by a one-digit number using formal written layout.	Multiply and divide numbers mentally drawing upon known facts.  Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000. Divide numbers by up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.  Solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign.	Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context.  Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context.

Fractions	<p>Recognise, find and name a half as one of two equal parts of an object, shape or quantity.</p> <p>Recognise, find and name a quarter as one of four equal parts of an object, shape or quantity</p>	<p>Recognise, find, name and write fractions <math>\frac{1}{3}</math>, <math>\frac{1}{4}</math>, <math>\frac{2}{4}</math> and <math>\frac{3}{4}</math> of a length, shape, set of objects or quantity.</p>	<p>Understand that fractions form part of a whole</p> <p>Make links between tenths, decimal measures and place value</p> <p>Understand the relationship between the numerator and the denominator</p> <p>Recognise and show equivalent fractions with different denominators</p> <p>Add and subtract fractions with the same denominator Order fractions with the same denominator]</p>	<p>Recognise and show, using diagrams, families of common equivalent fractions.</p> <p>Recognise and write decimal equivalents of any number of tenths or hundredths. Recognise and write decimal equivalents to <math>\frac{1}{4}</math>, <math>\frac{1}{2}</math>, <math>\frac{3}{4}</math>.</p> <p>Find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths.</p>	<p>Compare and order fractions whose denominators are all multiples of the same number. Identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths.</p> <p>Recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements greater than 1 as a mixed number [for example, <math>\frac{2}{5} + \frac{4}{5} = \frac{6}{5} = 1 \frac{1}{5}</math>].</p> <p>Add and subtract fractions with the same denominator and denominators that are multiples of the same number. Read and write decimal numbers as fractions [for example, <math>0.71 = \frac{71}{100}</math>].</p> <p>Recognise the per cent symbol (%) and understand that per cent relates to 'number of parts per hundred', and write percentages as a fraction with denominator 100, and as a decimal.</p>	<p>Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions.</p> <p>Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions.</p> <p>Multiply simple pairs of proper fractions, writing the answer in its simplest form [for example, <math>\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}</math>]. Divide proper fractions by whole numbers [for example, <math>\frac{1}{3} \div 2 = \frac{1}{6}</math>].</p> <p>Associate a fraction with division and calculate decimal fraction equivalents [for example, <math>0.375</math>] for a simple fraction [for example, <math>\frac{3}{8}</math>].</p> <p>Multiply one-digit numbers with up to two decimal places by whole numbers.</p> <p>Solve problems involving the calculation of percentages [for example, of measures, and such as 15% of 360] and the use of percentages for comparison.</p>



Measure		Choose and use appropriate standard units to estimate and measure length/height in any direction (m/cm); mass (kg/g); temperature (°C); capacity (litres/ml) to the nearest appropriate unit, using rulers, scales, thermometers and measuring vessels.	Know the appropriate unit of measurement and be able to use it to solve a problem Length – m, cm, mm Mass – kg, g Volume – l, ml Decide which is the appropriate unit of measure for the task set	Solve simple measure and money problems involving fractions and decimals to two decimal places.	Convert between different units of metric measure (for example, kilometre and metre; centimetre and metre; centimetre and millimetre; gram and kilogram; litre and millilitre).	Solve problems involving the calculation and conversion of units of measure, using decimal notation up to three decimal places where appropriate.
		Find different combinations of coins that equal the same amounts of money.  Tell and write the time to fifteen minutes, including quarter past/to the hour and draw the hands on a clock face to show these.	Understand, read and compare scales  Understand the value and equivalence of different coins and use this to solve mathematical problems	Convert between different units of measure [for example, kilometre to metre; hour to minute.	Measure and calculate the perimeter of composite rectilinear shapes in centimetres and metres. Solve problems involving converting between units of time.	Use, read, write and convert between standard units, converting measurements of length, mass, volume and time from a smaller unit of measure to a larger unit, and vice versa, using decimal notation to up to three decimal places.
Algebra, Geometry and Statistics	Recognise and name common 2-D and 3-D shapes, including 2-D shapes [for example, rectangles (including squares), circles and triangles]. Recognise and name common 2-D and 3-D shapes, including 3-D shapes [for example, cuboids (including cubes), pyramids and spheres	Identify and describe the properties of 2 -D shapes, including the number of sides and line symmetry in a vertical line.  Identify and describe the properties of 3 -D shapes, including the number of edges, vertices and faces	Name 2-D and 3-D shapes and describe their properties, including the angles  Interpret and present data using bar charts, pictograms and tables	Compare and classify geometric shapes, including quadrilaterals and triangles, based on their properties and sizes.  Identify acute and obtuse angles and compare and order angles up to two right angles by size.	Know angles are measured in degrees: estimate and compare acute, obtuse and reflex angles.  Solve comparison, sum and difference problems using information presented in a line graph.  Complete, read and interpret information in tables, including timetables.	Use simple formulae.  Find pairs of numbers that satisfy an equation with two unknowns.  Compare and classify geometric shapes based on their properties and sizes and find unknown angles in any triangles, quadrilaterals, and regular polygons.  Recognise angles where they meet at a point, are on a straight line, or are vertically opposite, and find missing angles.

## Appendix 4: Overview and Guidance

Planning and Resources	Teaching and Learning	Marking and Feedback
<ul style="list-style-type: none"> <li>White Rose Schemes of Learning – the ‘Small Steps’ provide a detailed sequence of T&amp;L and can be used with the support of ‘Power Maths’ and Abacus textbooks.</li> <li>These will be used to plan a continuum of learning through ‘Fluency, Reasoning and Problem Solving’</li> <li>Concrete, including the use of PV counters</li> <li>Shared Files on the One Drive contain a wealth of supporting resources for all year groups and objectives.</li> </ul>	<ul style="list-style-type: none"> <li>Teaching for Mastery: Whole class, mixed attaining</li> <li>Our priorities this year will be to develop our pedagogy for the teaching of mastery.</li> <li>To develop children’s knowledge, skills for reasoning and problem solving – including the routine use of spoken language structures to promote mastery.</li> <li>Bar Modelling</li> <li>Planned CPD : CPA, Mathematical Talk and Questioning, Planning for Depth and Reasoning and Problem Solving.</li> <li>Peer observations</li> <li>Working collaboratively with the 4LC and the White Rose Working Group</li> </ul>	<ul style="list-style-type: none"> <li><b><u>Efficient, meaningful and motivational</u></b></li> <li><b><u>Check/Correct, Consolidate, Challenge</u></b></li> <li><i>School policy: one deep mark per week or at assessment points</i></li> <li>Verbal or self assessed</li> <li>Promote independence</li> <li>Ensure children respond to feedback</li> <li>There are lots of examples provided by objective for extending and deepening:</li> <li>White Rose Guidance and examples</li> <li>NCETM Mastery documents</li> <li>Deepening Learning labels (on the One Drive)</li> <li>STEM Website</li> </ul>
Assessment	Working Walls	Interventions
<ul style="list-style-type: none"> <li>Key objectives in front of books are updated at the end of each block – children complete a mastery challenge to check understanding and gaps.</li> <li>Use of ‘Launch Pads’ (or quick assessment equivalent) for pre and post objectives assessment – these are analysed and used as your benchmark for planning/next steps.</li> <li>Summative assessments – WR have published these by term/year group. There is no threshold as they say they are to identify gaps and progress. However, Y6 will moderate these at the end of Autumn 1 and provide guidance then.</li> </ul>	<ul style="list-style-type: none"> <li><b><u>What are they for?</u></b></li> <li>Silent teacher – for children to reference when working independently.</li> <li>Reflect the current learning block/objectives.</li> <li>NOT – a maths display board</li> <li><b><u>What should they include?</u></b> (examples attached)</li> <li>Child friendly objectives for the block of learning</li> <li>Fluency – Reasoning – Problem solving sequence</li> <li>(Bronze - Silver - Gold)</li> <li>Worked examples modelled by teachers and some annotated examples from children.</li> <li>Tips and techniques (quotes from children)</li> <li>Anything else that would prompt and clarify for children who need it</li> <li>Opportunities for children at GD to deepen their understanding</li> </ul>	<ul style="list-style-type: none"> <li>To be decided by Leaders of Learning and Class Teachers and would normally take two forms:</li> <li><b><u>Fluency</u></b> – routine, regular learning and consolidation of Times Tables and arithmetic fluency</li> <li><b><u>Gaps</u></b> – quick response to marking or Launchpad pre/post assessment. Children identified by class teacher and work to correct or consolidate – these groups/interventions could include those children working at GD to challenge them to explain their concepts/methods to those who are working towards the objective. They should take place as near to the lesson as possible, E.g, during assembly.</li> </ul>

Fluency	Reasoning	Problem Solving	Arithmetic
<p>Key Vocabulary</p> <p>Large worked examples</p> <p>Factual fluency – recall facts</p> <p>Procedural fluency – worked examples</p> <p>Concrete and pictorial</p> <ul style="list-style-type: none"> <li>First</li> <li>I.....Then.....Next...Finally....</li> <li>I can show I am right because...</li> <li>This is wrong because...Therefore, the correct method would be...</li> </ul>	<p>Sentence Stems</p> <p>Worked Examples</p> <p>Abstract</p> <ul style="list-style-type: none"> <li>How many ways...?</li> <li>True or false? How do you know?</li> <li>Complete the statements to make them true</li> <li>What are the rules when (rounding)...</li> <li>When is it best to use this method?</li> <li>Draw me a picture to show that...</li> <li>What are the exceptions?</li> <li>Explain how you know that — is inaccurate...</li> <li>Explain why this happens...</li> <li>Always, sometimes, never...explain</li> </ul>	<p>Sentence stems</p> <p>‘Problem of the Week’</p> <p>‘Class Challenge’</p> <p>Which Problem Solving Strategy would you use?</p> <p>Show me how...</p> <p>Go APE:</p> <p>Answer</p> <p>Prove</p> <p>Explain</p> <ul style="list-style-type: none"> <li>Is there more than one option? Can you find them all?</li> <li>Write a story to represent your model/explanation...</li> <li>What is the greatest/least possible difference...?</li> <li>Can you work out.....explain your choices....</li> </ul> <p>You have mastered.....</p>	<p>Large worked examples of common errors</p> <p>Explain to _____ where they went wrong</p>